Consumption Partial Insurance in the Presence of Tail Income Risk*

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Introduction

- How does consumption respond to income shocks?
  - important for understanding how families cope with adverse events
  - important for designing government insurance, etc
- Household incomes are risky/uncertain, they fluctuate
- Extent to which **income fluctuations pass through to consumption** matters (we evaluate welfare on basis of consumption)

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- Household incomes are risky/uncertain, they fluctuate
- Extent to which **income fluctuations pass through to consumption** matters (we evaluate welfare on basis of consumption)
- Recent evidence from multiple countries shows that distribution of income risk has a **long left fat tail**
  - high probability of really bad income shocks

This paper

- **Measures consumption response to income shocks**, when distribution of said shocks exhibits skewness and kurtosis

Consumption partial insurance

- Blundell, Pistaferri, Preston, 2008 –abrv. **BPP**– introduce methodology to measure **pass-through** of income shocks to consumption
- BPP use US data from PSID and CEX in 1980-92:
  - *partial pass-through* of permanent income shocks
  - *no pass-through* of transitory income shocks

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  - *partial pass-through* of permanent income shocks
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- To obtain results, BPP rely on:
  - **log-linear consumption function** in income, i.e., linear transmission of income shocks
  - **workhorse covariance** between consumption and income growth, a second moment of their joint distribution
- Benchmark method and results in macro and labor economics

Income dynamics

- Guvenen et al, 2021 use admin data from US Social Sec. Administration and establish that the distribution of income growth is not log-normal:
  1. income growth is **very negatively skewed**
     $\hookrightarrow$ *far more people in the data with extreme negative than positive shocks*

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  2. income growth exhibits **very high kurtosis**
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- Benchmark results in literature, now seen in many countries
- Intuitively, income skewness and kurtosis should matter for pass-through of shocks to consumption
- Benchmark *linear transmission* method abstracts from these features
  $\rightarrow$ this is precisely what we address in this paper

What we do

1. **Re-estimate BPP** (1980-92 originally) in recent years (1999-2019)
   - maintain **linear transmission**, target income-consumption **second moms.**
   - while BPP imputed consumption from CEX into PSID, now consumption panel data available internally in PSID

2. **Target 3rd & 4th moments** in linear transmission model
   - go **beyond workhorse covariance** of income-consumption while maintaining **linear transmission**

3. **Generalize method of measuring partial insurance**
   - introduce higher-order consumption function that permits the **nonlinear transmission** of income risk
   - establish identification in generalized model
   - characterize and measure consumption pass-through of income shocks
   - quantify welfare implications of tail income risk

What we find households aged 30-65 in PSID, 1999-2019

1. **Re-estimate BPP**: much lower pass-through of permanent shocks
   - **0.15 in recent years**, vs. 0.64 originally
   - imputation of consumption from CEX & higher frequency of data in older PSID inflate pass-through

2. Target **3rd & 4th moments** within linear model: pass-through stays small
   - unsurprising: tail risk has no role in linear model

3. Generalize to **higher-order consumption function**:
   - bad permanent shocks more unsettling than good ones
   - consider a 3σ permanent shock:
     $\hookrightarrow$ a 50% income cut $\rightarrow$ pass-through 17%
     $\hookrightarrow$ a 50% income gain $\rightarrow$ pass-through 9%

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     $\hookrightarrow$ pass-through depends on the sign and size of the shock
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   - bad shocks more hurtful among the rich due to nature of income risk, not due to consumption technology
   - welfare cost of tail risk: 1/8 of lifetime consumption

Literature

1. **Consumption-income link** with two distinct branches
   i. **fully specified models** model specific insurance channels

      Kaplan & Violante, 2010; Guvenen & Smith, 2014; Heathcote et al, 2014; Blundell et al, 2018; De Nardi et al, 2019; Madera, 2019; Krueger & Wu, 2022; Busch & Ludwig, 2023; Guvenen et al, 2024

   ii. **partially specified models** measure overall partial insurance

      Blundell, Pistaferri, Preston, 2008; Blundell et al, 2016; Arellano et al, 2017; Crawley, 2020; Theloudis, 2021; Chopra, 2022; Commault, 2022; Hryshko & Manovskii, 2022; Arellano et al, 2023

2. **Modern income dynamics**

   Geweke & Keane, 2000; Bonhomme & Robin, 2010; Guvenen et al, 2014; Guvenen et al, 2021; De Nardi et al, 2021; Busch et al, 2022; QE Global Income Dynamics 2022 issue

3. **Elicit responses to (quasi-)experimental events / MPCs**

   Johnson et al, 2006; Parker at al, 2013; Christelis et al, 2019; Fuster et al, 2020; Jappelli & Pistaferri, 2020; Fagereng et al, 2021; Lewis et al, 2022; Crawley and Kuchler, 2023; Gelman et al, 2023

4. **Role of nonlinearities and/or tail risk**

   Contantinides & Ghosh, 2017; McKay, 2017; Baqaee & Farhi, 2019; Ai & Bhandari, 2021; Catherine et al, 2023

- **Our contribution:** measurement paper like 1ii, accounting for facts in 2, thus also contributing to 3 & 4

0.
Lifecycle consumption model with general income risk

Household problem

- Household i chooses sequence of consumption C_{it} and savings A_{it+1} to maximize expected lifetime utility

$$\max_{\{C_{it}, A_{it+1}\}_{t=0}^{T}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t U(C_{it}; \mathbf{Z}_{it})$$

subject to budget constraint and terminal condition:

$$A_{it} + Y_{it} = C_{it} + (1+r)^{-1} A_{it+1}, \qquad \forall t \in \{0, \ldots, T\}$$
$$A_{iT+1} = 0$$

▸ model solution

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- Preferences:
  - $U(\cdot)$ utility over consumption, well behaved, unitary household
  - $\mathbf{Z}_{it}$ deterministic taste shifters
- Budget constraint:
  - Y_{it} stochastic **household disposable income**
  - r deterministic interest rate

▸ model solution

Income process

- Today: **permanent-transitory** process for log income

$$\begin{aligned} \ln Y_{it} &= \mathbf{X}_{it}'\boldsymbol{\delta} + P_{it} + v_{it} \\ P_{it} &= P_{it-1} + \zeta_{it} \end{aligned} \tag{1}$$

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$$P_{it} = P_{it-1} + \zeta_{it}$$

$$\Delta y_{it} = \overbrace{\zeta_{it}}^{\text{permanent shock}} + \underbrace{\Delta v_{it}}_{\substack{\text{change in} \\ \text{transitory shock}}} \tag{1}$$

  where $\Delta y_{it} = \Delta \ln Y_{it} - \Delta \mathbf{X}'_{it}\boldsymbol{\delta}$ stochastic component of income

- Shocks are *i.i.d.* across households within t (risk is idiosyncratic)

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- Shocks are *i.i.d.* across households within t (risk is idiosyncratic)
- Paper presents & estimates extension to process with **linear persistence**

Income process

stochastic properties

- Permanent & transitory shocks are **non-Gaussian** with first 4 cross-sectional moments given by:

$$\mathbb{E}(\zeta_{it}^m) = \begin{cases} 0 \\ \sigma^2_{\zeta_t} \\ \gamma_{\zeta_t} \\ \kappa_{\zeta_t} \end{cases} \quad \text{and} \quad \mathbb{E}(v_{it}^m) = \begin{cases} 0 & \text{for } m = 1 \\ \sigma^2_{v_t} & \text{for } m = 2 \\ \gamma_{v_t} & \text{for } m = 3 \\ \kappa_{v_t} & \text{for } m = 4 \end{cases}$$

- Moments:
  - σ^2 measures **dispersion** (2^{nd} moment)
  - γ measures **skewness** (3^{rd} moment, length of tail)
  - κ measures **kurtosis** (4^{th} moment, thickness of tail)
- Shocks are independent over time, and $\zeta_{it} \perp v_{j\tau}$ for all i, j and t, τ

1.
Replication of BPP
in recent period

Linear consumption function

- BPP assume higher-order moments away and **log-linearize**

  (1.) problem's first-order conditions,
  (2.) intertemporal budget constraint,

  and get the **linear consumption function**:

$$\Delta c_{it} = \xi_{it} + \phi_{it}^{(1)} \zeta_{it} + \psi_{it}^{(1)} v_{it} \tag{2}$$

  where Δc_{it} is consumption growth net of taste shifters $\mathbf{Z}_{it}$

- Unobserved taste heterogeneity ξ_{it}, independent of shocks

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- **Linear transmission of income shocks**:

	pass-through	degree of partial insurance
permanent:	$\phi_{it}^{(1)} = \partial \Delta c_{it} / \partial \zeta_{it}$	
transitory:	$\psi_{it}^{(1)} = \partial \Delta c_{it} / \partial v_{it}$	

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permanent:	$\phi_{it}^{(1)} = \partial \Delta c_{it} / \partial \zeta_{it}$	$1 - \phi_{it}^{(1)}$
transitory:	$\psi_{it}^{(1)} = \partial \Delta c_{it} / \partial v_{it}$	$1 - \psi_{it}^{(1)}$

Identification in linear function

- Identification (assuming $_{it}$ invariant) with **second moments** only:

$$\phi^{(1)} = \mathrm{Cov}(\Delta c_{it}, \overline{\Delta y_{it}})/\mathrm{Cov}(\Delta y_{it}, \overline{\Delta y_{it}})$$
$$\psi^{(1)} = \mathrm{Cov}(\Delta c_{it}, \Delta y_{it+1})/\mathrm{Cov}(\Delta y_{it}, \Delta y_{it+1})$$

where $\overline{\Delta y_{it}} = (1/3)\sum_{\tau=-1}^{1} \Delta y_{it+\tau}$ is permanent income at t

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- Identification requires **panel data** in
  - income $\rightarrow$ available in PSID
  - consumption $\rightarrow$ no consumption data in PSID before 1999

- BPP resort to imputation of consumption from CEX into PSID, based on commonly available food expenditure

Results: linear model

- Estimate linear model over 1999-2019; consumption available internally in PSID

	Original period 1980-92
Income moments:	2^{nd}
Consumption data:	imputed CEX
Data frequency:	*annually*
	(3)
$\phi^{(1)}$ (permanent)	0.642
	(0.095)
$\psi^{(1)}$ (transitory)	0.053
	(0.044)
σ^2_{ξ} (taste heterog.)	0.011
	(0.004)
$\sigma^2_{u_c}$ (cons. measur. error)	–
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Income moments:	2^{nd}	2^{nd}
Consumption data:	PSID	imputed CEX
Data frequency:	*biennially*	*annually*
	(1)	(3)
$\phi^{(1)}$ (permanent)	0.152	0.642
	(0.028)	(0.095)
$\psi^{(1)}$ (transitory)	-0.006	0.053
	(0.045)	(0.044)
σ^2_ξ (taste heterog.)	0.019	0.011
	(0.001)	(0.004)
$\sigma^2_{u_c}$ (cons. measur. error)	0.044	–
	(0.002)	–

  - much **lower pass-through** of permanent shocks than BPP
  - low pass-through seen in, e.g., Arellano et al, 2017/23, for similar years

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Data frequency:	*biennially*	*biennially*	*annually*
	(1)	(2)	(3)
$\phi^{(1)}$ (permanent)	0.152	0.288	0.642
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$\psi^{(1)}$ (transitory)	-0.006	-0.109	0.053
	(0.045)	(0.078)	(0.044)
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$\phi^{(1)}$ (permanent)	0.152	0.288	0.642	0.404
	(0.028)	(0.044)	(0.095)	(0.057)
$\psi^{(1)}$ (transitory)	-0.006	-0.109	0.053	0.158
	(0.045)	(0.078)	(0.044)	(0.051)
σ^2_ξ (taste heterog.)	0.019	0.019	0.011	0.013
	(0.001)	(0.004)	(0.004)	(0.003)
$\sigma^2_{u_c}$ (cons. measur. error)	0.044	0.140	–	0.070
	(0.002)	(0.009)	–	(0.005)

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  - imputation from CEX explains 1/3 of difference
  - survey design effects explain another 1/3 of difference
  - we attribute remaining gap to calendar time effects (Borella et al, 2022)

2.
Introduction of higher-order moments in the linear framework

Skewness and kurtosis in the PSID

- Guvenen et al's income facts prevalent also in our data
  - income growth exhibits **large left skewness** and **very high kurtosis**
  - PSID income moments very close to administrative moments
- Consumption growth *less* skewed or leptokurtic, but *not* log-normal
  - no administrative targets to benchmark consumption moments against

▸ main moments ▸ more

Targeting skewness and kurtosis in linear model

- Using linear income and consumption processes

$$\Delta y_{it} = \zeta_{it} + \Delta v_{it} \tag{1}$$

$$\Delta c_{it} = \xi_{it} + \phi_{it}^{(1)} \zeta_{it} + \psi_{it}^{(1)} v_{it} \tag{2}$$

  we construct theoretical 2^{nd}, 3^{rd}, 4^{th} moments of joint distribution of income and consumption

  - then match to empirical ones
  - and estimate $\phi^{(1)}$ and $\psi^{(1)}$ using **full information** on all moments

▸ all targets

Limitations of linear transmission model

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  - adequate approximation to true consumption rule when shocks do not move household too far away from deterministic profile

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- Targeting higher-order moments within the linear consumption model does not change transmission parameters
  - there now seems to be slightly *more* insurance to permanent shocks
- Linear transmission model relies on the **log-linearization** of the consumption rule around a deterministic lifecycle profile
  - adequate approximation to true consumption rule when shocks do not move household too far away from deterministic profile
- But tail shocks may command large shifts in consumption as a consequence of concave utility
  - they may move household far away from deterministic profile
  - **linear transmission model** provides poor approximation in the presence of this type of shocks
  - echoes Carroll 2001's critique of log-linearized Euler equations
- Linear consumption function misspecified in the presence of tail risk

3.
Generalization to nonlinear transmission

Nonlinear consumption function

- We generalize method by carrying out a **second-order approximation** to (1.) problem's first-order conditions, (2.) intertemp. budget constraint

▸ approximation details

Nonlinear consumption function

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- We obtain the **quadratic consumption function**:

$$\Delta c_{it} = \xi_{it} + \underbrace{\phi_{it}^{(1)}\zeta_{it} + \psi_{it}^{(1)}v_{it}}_{\text{BPP's linear part}} + \underbrace{\phi_{it}^{(2)}\zeta_{it}^{2} + \psi_{it}^{(2)}v_{it}^{2} + \omega_{it}^{(22)}\zeta_{it}v_{it}}_{\text{nonlinear part}} \tag{3}$$

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- Parameters:
  - linear transmission parameters: $\phi_{it}^{(1)}$, $\psi_{it}^{(1)}$
  - non-linear transmission parameters: $\phi_{it}^{(2)}$, $\psi_{it}^{(2)}$, $\omega_{it}^{(22)}$
  - unobserved taste heterogeneity ξ_{it}

▸ approximation details

Nonlinear transmission

- Pass-through of permanent shocks, given by

$$\begin{aligned} \text{BPP/linear}: &\quad \partial \Delta c_{it} / \partial \zeta_{it} = \phi^{(1)} \\ \text{Nonlinear}: &\quad \partial \Delta c_{it} / \partial \zeta_{it} = \phi^{(1)} + 2\phi^{(2)} \zeta_{it} + \omega^{(22)} v_{it} \end{aligned} \tag{4}$$

- (4) depends on $\phi^{(1)}$, $\phi^{(2)}$, $\omega^{(22)}$, and sign and magnitude of shocks

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- Let $\phi^{(1)} > 0$ and $\phi^{(2)} < 0$:
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- (4) depends on $\phi^{(1)}$, $\phi^{(2)}$, $\omega^{(22)}$, and sign and magnitude of shocks
- Let $\phi^{(1)} > 0$ and $\phi^{(2)} < 0$:
  $\hookrightarrow$ a bad ζ_{it} has larger impact on consumption than a good ζ_{it}
- Workhorse covariance of consumption-income:
  - does not identify true transmission parameter
  - **biased** even for $\phi^{(1)}$ if permanent shocks have skewness

$$\frac{\mathrm{Cov}(\Delta c_{it}, \overline{\Delta y_{it}})}{\mathrm{Cov}(\Delta y_{it}, \overline{\Delta y_{it}})} = \phi^{(1)} + \underbrace{\phi^{(2)} \frac{\gamma_{\zeta_t}}{\sigma^2_{\zeta_t}}}_{\text{bias}}$$

Identification

- We need to identify:
  - moments of income shocks $\sigma^2_{\zeta_t}$, $\sigma^2_{v_t}$, γ_{ζ_t}, γ_{v_t}, κ_{ζ_t}, κ_{v_t}
  - linear transmission parameters $\phi^{(1)}_{it}$, $\psi^{(1)}_{it}$
  - nonlinear transmission parameters $\phi^{(2)}_{it}$, $\psi^{(2)}_{it}$, $\omega^{(22)}_{it}$
  - variance of taste shocks $\sigma^2_{\xi_t}$ and consumption measurement error
- We **establish identification** for **any** distribution of income shocks, **except** Bernoulli $\rightarrow$ main assumption: existence of moments

identification y | identification c | target moments

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- We **establish identification** for **any** distribution of income shocks, **except** Bernoulli $\rightarrow$ main assumption: existence of moments
- We establish identification with:
  - 2^{nd}, 3^{rd}, 4^{th} moments of income
  - and 2^{nd} moments of consumption only

identification y | identification c | target moments

Identification

- We need to identify:
  - moments of income shocks $\sigma^2_{\zeta_t}$, $\sigma^2_{v_t}$, γ_{ζ_t}, γ_{v_t}, κ_{ζ_t}, κ_{v_t}
  - linear transmission parameters $\phi^{(1)}_{it}$, $\psi^{(1)}_{it}$
  - nonlinear transmission parameters $\phi^{(2)}_{it}$, $\psi^{(2)}_{it}$, $\omega^{(22)}_{it}$
  - variance of taste shocks $\sigma^2_{\xi_t}$ and consumption measurement error
- We **establish identification** for **any** distribution of income shocks, **except** Bernoulli $\rightarrow$ main assumption: existence of moments
- We establish identification with:
  - 2^{nd}, 3^{rd}, 4^{th} moments of income
  - and 2^{nd} moments of consumption only
- We do **not** use higher-order moments of consumption
  - unlike income moments, we cannot benchmark consumption moments against administrative sources
  - we find meaningful differences in **higher-order consumption moments** between PSID and CEX $\rightarrow$ we should worry!

▸ identification y ▸ identification c ▸ target moments

Results: income process

Income moments:			2^{nd}		2^{nd}, 3^{rd}		2^{nd}, 3^{rd}, 4^{th}	
Income data:			PSID		PSID		PSID	
			(1)		(2)		(3)	
Panel A. Permanent shocks								
Variance	σ_ζ^2		0.027	(0.002)	0.027	(0.002)	0.030	(0.002)
Skewness	γ_ζ	central			-0.004	(0.003)	-0.004	(0.003)
		standardized			-0.947	(0.649)	-0.831	(0.571)
Kurtosis	κ_ζ	central					0.039	(0.007)
		standardized					44.393	(11.080)
Panel B. Transitory shocks								
Variance	σ_v^2		0.033	(0.002)	0.033	(0.002)	0.031	(0.003)
Skewness	γ_v	central			-0.008	(0.003)	-0.008	(0.003)
		standardized			-1.310	(0.510)	-1.431	(0.566)
Kurtosis	κ_v	central					0.048	(0.005)
		standardized					49.226	(10.119)

Estimation is via equally weighted GMM; block bootstrap standard errors are in parentheses. Results from optimal GMM are similar.

- **Long left tail** for both shocks, **super large kurtosis**
  $\hookrightarrow$ so most income shocks either zero, or really very bad!
- Sources of bad income shocks: *unemployment*, *health*, *productivity*, ...

Results: nonlinear model

Consumption fn.:	Linear			**Quadratic**
Income moments:	2^{nd}	2^{nd}, 3^{rd}	2^{nd}, 3^{rd}, 4^{th}	2^{nd}, 3^{rd}, 4^{th}
Consumption data:	PSID	PSID	PSID	PSID
	(1)	(2)	(3)	(4)
$\phi^{(1)}$	0.152	0.160	0.112	0.134
	(0.028)	(0.029)	(0.041)	(0.026)
$\psi^{(1)}$	-0.006	-0.012	0.007	**-0.003**
	(0.045)	(0.038)	(0.056)	(0.052)
$\phi^{(2)}$				-0.040
				(0.026)
$\psi^{(2)}$				**0.015**
				(0.034)
$\omega^{(22)}$				0.320
				(0.770)
σ^2_ξ (taste heterog.)	0.019	0.019	0.020	0.019
	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma^2_{u_c}$ (cons. measur. error)	0.044	0.044	0.044	0.044
	(0.002)	(0.002)	(0.001)	(0.002)

Estimation is via equally weighted GMM; block bootstrap standard errors are in parentheses.
Results from optimal GMM are similar.

- **Transitory shocks:** no pass-through to consumption

Results: nonlinear model

Consumption fn.:	Linear			**Quadratic**
Income moments:	2^{nd}	2^{nd}, 3^{rd}	2^{nd}, 3^{rd}, 4^{th}	2^{nd}, 3^{rd}, 4^{th}
Consumption data:	PSID	PSID	PSID	PSID
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$\phi^{(2)}$				**-0.040**
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	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma^2_{u_c}$ (cons. measur. error)	0.044	0.044	0.044	0.044
	(0.002)	(0.002)	(0.001)	(0.002)

Estimation is via equally weighted GMM; block bootstrap standard errors are in parentheses.
Results from optimal GMM are similar.

- **Transitory shocks:** no pass-through to consumption
- **Permanent shocks:**
  - $\phi^{(1)} > 0$ (as one would expect)
  - $\phi^{(2)} < 0$ (new estimate)

Results: nonlinear transmission

- Let transitory shock be $v_{it} = 0$, then

$$\partial \Delta c_{it} / \partial \zeta_{it} = \phi^{(1)} + 2\phi^{(2)} \zeta_{it} = 0.134 - 0.08 \times \zeta_{it} \tag{5}$$

- **Two main results:**

Results: nonlinear transmission

- Let transitory shock be $v_{it} = 0$, then

$$\partial \Delta c_{it} / \partial \zeta_{it} = \phi^{(1)} + 2\phi^{(2)} \zeta_{it} = 0.134 - 0.08 \times \zeta_{it} \tag{5}$$

- **Two main results:**
  1. **negative shocks more hurtful than positive ones**

$$\left. \frac{\partial \Delta c_{it}}{\partial \zeta_{it}} \right|_{\zeta<0} > \left. \frac{\partial \Delta c_{it}}{\partial \zeta_{it}} \right|_{\zeta>0}$$

Results: nonlinear transmission

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$$\hookrightarrow \partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=-0.5} = 0.174 \text{ but } \partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=0.5} = 0.09$$

Results: nonlinear transmission

- Let transitory shock be $v_{it} = 0$, then

$$\partial \Delta c_{it} / \partial \zeta_{it} = \phi^{(1)} + 2\phi^{(2)} \zeta_{it} = 0.134 - 0.08 \times \zeta_{it} \tag{5}$$

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$\hookrightarrow \partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=-0.5} = 0.174$ but $\partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=0.5} = 0.09$

  2. **larger bad shocks more hurtful than smaller ones**

$$\left. \frac{\partial \Delta c_{it}}{\partial \zeta_{it}} \right|_{\zeta<<<0} > \left. \frac{\partial \Delta c_{it}}{\partial \zeta_{it}} \right|_{\zeta<0}$$

$\hookrightarrow \partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=-0.75} = 0.194$ but $\partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=-0.30} = 0.158$

Results: by age

Consumption fn.:	**Quadratic**					
Age household head:	30–40		41–50		51–65	
Income moments:	2^{nd}, 3^{rd}, 4^{th}		2^{nd}, 3^{rd}, 4^{th}		2^{nd}, 3^{rd}, 4^{th}	
Consumption data:	PSID		PSID		PSID	
	(1)		(2)		(3)	
$\phi^{(1)}$	**0.149**	(0.050)	**0.115**	(0.049)	**0.113**	(0.057)
$\psi^{(1)}$	0.105	(0.090)	0.019	(0.090)	-0.063	(0.085)
$\phi^{(2)}$	**0.006**	(0.038)	**-0.008**	(0.063)	**-0.104**	(0.052)
$\psi^{(2)}$	0.036	(0.061)	0.048	(0.046)	-0.052	(0.073)
$\omega^{(22)}$	-1.089	(1.465)	-0.294	(1.195)	2.984	(0.791)
σ^2_ξ	0.013	(0.004)	0.022	(0.002)	0.010	(0.005)
$\sigma^2_{u_c}$	0.046	(0.003)	0.040	(0.002)	0.043	(0.002)

Estimation is via equally weighted GMM; block bootstrap standard errors are in parentheses.

- Quadratic function: decreasing pattern in both $\phi^{(1)}$ and $\phi^{(2)}$
  - **bad & large shocks more hurtful at older ages**, have larger pass-through $\hookrightarrow \partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=-0.5,\text{age}=45} = 0.123$ but $\partial \Delta c_{it} / \partial \zeta_{it}|_{\zeta=-0.5,\text{age}=60} = 0.218$
  - Guvenen et al, 2021: income skewness more negative w. age & earnings, more room to "fall" and cause harm

Results: by position in income distribution

Consumption fn.:	**Quadratic**			
Lag income y_{it-1}*:*	Bottom tertile **'poor'**		Top tertile **'rich'**	
Income moments:	2^{nd}, 3^{rd}, 4^{th}		2^{nd}, 3^{rd}, 4^{th}	
Consumption data:	PSID		PSID	
	(1)		(2)	
$\phi^{(1)}$	**0.113**	(0.029)	**0.063**	(0.100)
$\psi^{(1)}$	0.186	(0.199)	0.275	(0.200)
$\phi^{(2)}$	**-0.064**	(0.029)	**-0.025**	(0.101)
$\psi^{(2)}$	0.210	(0.220)	-0.268	(0.197)
$\omega^{(22)}$	1.965	(1.513)	1.072	(2.144)
σ^2_ξ	0.011	(0.005)	0.025	(0.004)
$\sigma^2_{u_c}$	0.044	(0.002)	0.041	(0.002)

Estimation is via equally weighted GMM; standard errors clustered at the household level in parentheses.

1. $\phi^{(1)}_{\text{rich}} < \phi^{(1)}_{\text{poor}} \rightarrow$ the poor have less insurance to average shock

Results: by position in income distribution

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Lag income y_{it-1}*:*	Bottom tertile **'poor'**		Top tertile **'rich'**	
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Consumption data:	PSID		PSID	
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2. $\phi^{(2)}_{\text{rich}} > \phi^{(2)}_{\text{poor}} \rightarrow$ the poor have less insurance to *bad* shock of *given size*
   - but nature of income risk very different between poor and rich: $skew(\zeta_{it})_{\text{poor}} = +1.34$ but $skew(\zeta_{it})_{\text{rich}} = -4.86$

Results: by position in income distribution

Consumption fn.:	**Quadratic**			
Lag income y_{it-1}*:*	Bottom tertile **'poor'**		Top tertile **'rich'**	
Income moments:	2^{nd}, 3^{rd}, 4^{th}		2^{nd}, 3^{rd}, 4^{th}	
Consumption data:	PSID		PSID	
	(1)		(2)	
$\phi^{(1)}$	**0.113**	(0.029)	**0.063**	(0.100)
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   - but nature of income risk very different between poor and rich: $skew(\zeta_{it})_{\text{poor}} = +1.34$ but $skew(\zeta_{it})_{\text{rich}} = -4.86$

   $\Rightarrow$ bad ζ_{it} from **given position** in underlying income distribution shifts consumption **more strongly downwards** among the rich

Results: by position in income distribution

Consumption fn.:	**Quadratic**			
Lag income y_{it-1}*:*	Bottom tertile **'poor'**		Top tertile **'rich'**	
Income moments:	2^{nd}, 3^{rd}, 4^{th}		2^{nd}, 3^{rd}, 4^{th}	
Consumption data:	PSID		PSID	
	(1)		(2)	
$\phi^{(1)}$	**0.113**	(0.029)	**0.063**	(0.100)
$\psi^{(1)}$	0.186	(0.199)	0.275	(0.200)
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σ^2_ξ	0.011	(0.005)	0.025	(0.004)
$\sigma^2_{u_c}$	0.044	(0.002)	0.041	(0.002)

Estimation is via equally weighted GMM; standard errors clustered at the household level in parentheses.

1. $\phi^{(1)}_{\text{rich}} < \phi^{(1)}_{\text{poor}} \rightarrow$ the poor have less insurance to average shock
2. $\phi^{(2)}_{\text{rich}} > \phi^{(2)}_{\text{poor}} \rightarrow$ the poor have less insurance to *bad* shock of *given size*
   - but nature of income risk very different between poor and rich: $skew(\zeta_{it})_{\text{poor}} = +1.34$ but $skew(\zeta_{it})_{\text{rich}} = -4.86$

   $\Rightarrow$ bad ζ_{it} from **given position** in underlying income distribution shifts consumption **more strongly downwards** among the rich
   - two forces at play: nature of income risk ✓, not consumption technology ✗

Results: by wealth and education

Consumption fn.:	**Quadratic**			
	Low wealth		High wealth	
	No college	Some college	No college	Some college
Income moments:	2^{nd}, 3^{rd}, 4^{th}	2^{nd}, 3^{rd}, 4^{th}	2^{nd}, 3^{rd}, 4^{th}	2^{nd}, 3^{rd}, 4^{th}
Consumption data:	PSID	PSID	PSID	PSID
	(1)	(2)	(3)	(4)
$\phi^{(1)}$	**0.242**	**0.192**	**0.164**	**0.079**
	(0.051)	(0.052)	(0.067)	(0.034)
$\psi^{(1)}$	-0.251	0.015	0.006	0.004
	(0.131)	(0.108)	(0.085)	(0.073)
$\phi^{(2)}$	**-0.094**	**0.005**	**-0.076**	**-0.032**
	(0.062)	(0.052)	(0.057)	(0.041)
$\psi^{(2)}$	-0.162	-0.001	0.177	0.001
	(0.087)	(0.074)	(0.089)	(0.048)
$\omega^{(22)}$	2.998	-0.660	-2.228	0.441
	(1.423)	(1.530)	(1.568)	(0.995)
σ^2_ξ	0.006	0.013	0.006	0.025
	(0.008)	(0.002)	(0.010)	(0.002)
$\sigma^2_{u_c}$	0.046	0.045	0.038	0.043
	(0.003)	(0.003)	(0.004)	(0.002)

Estimation is via equally weighted GMM; standard errors clustered at the household level in parentheses.

- $\phi^{(1)}$ decreases monotonically as we move left $L \rightarrow R$ right
  - **self-insurance** role of wealth, wealth increases monotonically $L \rightarrow R$
- $\phi^{(2)}$ turns less negative from left to right
- Transitory shocks v_{it} pass through among the least wealthy

More results in the paper

- Generalization to income process with AR(1) linear persistence
  - main consumption results maintained, statistical significance improved
- Simulation of income shocks and comparison of pass-through with quantile framework of Arellano, Blundell, Bonhomme (2017)
  - mostly similar consumption growth rates between two papers
  - nonlinear income persistence not crucial for main consumption results
- Welfare effects of tail risk in reasonably parametrized economy
  - households give up to 1/8 of lifetime consumption to avoid tail risk

Conclusions

Conclusions

1. We **re-estimate** Blundell, Pistaferri, Preston's linear transmission model using new consumption data in PSID in recent years
   - very low pass-through of permanent income shocks
2. We target **skewness & kurtosis moments** within linear model, to account for salient income facts in Guvenen et al, 2021
   - pass-through decreases slightly further
3. We generalize the benchmark linear transmission model to enable **nonlinear transmission** and a role for tail income risk
   - we establish identification of pass-through in such setting
   - larger & bad shocks more impactful than smaller & good ones

- **Tail income risk & non-linearities matter for consumption**

Appendix

A.1
Model appendix

Model solution

optimality conditions

- Let utility be $U(C_{it}; \mathbf{Z}_{it}) = \widetilde{U}(C_{it} \exp(-\mathbf{Z}'_{it}\boldsymbol{\alpha}))$
- We work with the first-order conditions (necessary & sufficient for optimality in this case):

$$\begin{aligned} [C_{it}]: \quad & \widetilde{U}_C(\widetilde{C}_{it}) \exp(-\mathbf{Z}'_{it}\boldsymbol{\alpha}) = \lambda_{it} \\ [A_{it+1}]: \quad & \beta(1+r)\mathbb{E}_t \lambda_{it+1} = \lambda_{it} \end{aligned}$$

where

  - $\widetilde{C}_{it} = C_{it} \exp(-\mathbf{Z}'_{it}\boldsymbol{\alpha})$
  - λ_{it} is the marginal utility of wealth

Model solution

approximations

1. Second-order approximation to **Euler equation**, combined with log-linearization of marginal utility $\widetilde{U}_C(\widetilde{C}_{it})$, yields

$$\Delta c_{it} \approx \xi_{it} + \theta_{it-1}\epsilon_{it}$$

   - Δc_{it} consumption net of taste observables and gradient due to r and β
   - ξ_{it} unobserved taste heterogeneity (higher moments of λ)
   - θ_{it} inverse of consumption substitution elasticity
   - ϵ_{it} expectation error, idiosyncratic revisions to λ due to income shocks

2. Second-order approximation to **intertemporal budget constraint**

$$\begin{aligned}\theta_{it-1}\epsilon_{it} \approx (1-\pi_{it})\zeta_{it} + (1-\pi_{it})\vartheta_{it}^{Y} v_{it} + \frac{1}{2}(1-\pi_{it})\pi_{it}\zeta_{it}^2 \\ + \frac{1}{2}(1-\pi_{it})\vartheta_{it}^{Y}(1-(1-\pi_{it})\vartheta_{it}^{Y})v_{it}^2 + (1-\pi_{it})\vartheta_{it}^{Y}\pi_{it}\zeta_{it}v_{it}\end{aligned}$$

   - π_{it} is share of financial wealth in total financial and human wealth at t
   - ϑ_{it}^{Y} is share of time t household income in total lifetime income

- BPP is a special case of above (**first-order** approximations only)

A.2
Consumption imputation

Consumption imputation

from CEX into PSID

- Let f_{it} be log real food expenditure at t, available in both PSID and CEX
- Consider demand equation for food

$$f_{it} = \mathbf{W}'_{it}\boldsymbol{\mu} + \mathbf{p}'_t\boldsymbol{\theta} + \beta[D_{it}]c_{it} + e_{it}$$

  - $\mathbf{W}_{it}$ contains demographic variables
  - $\mathbf{p}_t$ is vector of prices of food and other goods
  - $\beta[\cdot]$ is budget elasticity of food, changes with demographics D_{it}
  - c_{it} is log of nondurable expenditure, only available in CEX
- BPP estimate food demand on CEX, instrument c_{it} with income
- Under *monotonicity* of food demand, BPP invert equation, use it to impute c_{it} in the PSID

A.3
Identification

Identification

income process

- Permanent shocks:

$$\sigma_\zeta^2 = \mathbb{E}\left[\Delta y_{it} \times (\Delta y_{it-1} + \Delta y_{it} + \Delta y_{it+1})\right]$$
$$\gamma_\zeta = \mathbb{E}\left[(\Delta y_{it})^2 \times (\Delta y_{it-1} + \Delta y_{it} + \Delta y_{it+1})\right]$$
$$\kappa_\zeta = \mathbb{E}\left((\Delta y_{it})^4\right) - 2\kappa_v - 6(\sigma_v^2)^2 - 12\sigma_\zeta^2\sigma_v^2$$

- Transitory shocks:

$$\sigma_v^2 = -\mathbb{E}\left[\Delta y_{it} \times \Delta y_{it+1}\right]$$
$$\gamma_v = -\mathbb{E}\left[(\Delta y_{it})^2 \times \Delta y_{it+1}\right]$$
$$\kappa_v = \mathbb{E}\left((\Delta y_{it})^2 \times (\Delta y_{it+1})^2\right) - (\sigma_\zeta^2)^2 - 3(\sigma_v^2)^2 - 4\sigma_\zeta^2\sigma_v^2$$

- Adjustments must be made for income measurement error and biennial nature of PSID

Identification

quadratic consumption function

- The transmission parameters are given by

$$\left(\phi^{(1)}, \psi^{(1)}, \phi^{(2)}, \psi^{(2)}, \omega^{(22)}\right)' = \mathbf{A}^{-1}\mathbf{B}$$

with

$$\mathbf{A} = \begin{pmatrix} 0 & \sigma_v^2 & 0 & \gamma_v & 0 \\ 0 & \gamma_v & 4\sigma_\zeta^2\bar{\sigma} & \kappa_v + \sigma_v^2(2\bar{\sigma} - \sigma_v^2) & 0 \\ 2\sigma_\zeta^2 & \sigma_v^2 & 2\gamma_\zeta & \gamma_v & 0 \\ 2\gamma_\zeta & \gamma_v & 2(\kappa_\zeta + \sigma_\zeta^2(2\bar{\sigma} + \sigma_\zeta^2)) & \kappa_v + \sigma_v^2(2\bar{\sigma} - \sigma_v^2) & 4\sigma_\zeta^2\sigma_v^2 \\ 0 & \gamma_v & 2\sigma_\zeta^2(\bar{\sigma} - \sigma_\zeta^2) & \kappa_v + \sigma_v^2\sigma_{u^y}^2 & 2\sigma_\zeta^2\sigma_v^2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} -\mathbb{E}\left(\Delta c_{it} \times \Delta y_{it+1}\right) \\ \mathbb{E}\left(\Delta c_{it} \times (\Delta y_{it+1})^2\right) \\ \mathbb{E}\left(\Delta c_{it} \times \Delta y_{it}\right) \\ \mathbb{E}\left(\Delta c_{it} \times (\Delta y_{it})^2\right) \\ -\mathbb{E}\left(\Delta c_{it} \times \Delta y_{it} \times \Delta y_{it+1}\right) \end{pmatrix}$$

- Matrix **A** is full rank for **any distribution** of shocks, except Bernoulli
- Taste heterogeneity and consumption error are identified by the second moments of consumption

Target moments

- BPP (2^{nd} moments only):

$$\mathbb{E}((\Delta y_{it})^2) \quad \mathbb{E}(\Delta y_{it} \times \Delta y_{it-2}) \quad \mathbb{E}((\Delta c_{it})^2)$$
$$\mathbb{E}(\Delta c_{it} \times \Delta c_{it-2}) \quad \mathbb{E}(\Delta y_{it} \times \Delta c_{it}) \quad \mathbb{E}(\Delta y_{it} \times \Delta c_{it-2})$$

- $+3^{rd}$ moments:

$$\mathbb{E}((\Delta y_{it})^3) \quad \mathbb{E}((\Delta y_{it})^2 \times \Delta y_{it-2}) \quad \mathbb{E}(\Delta y_{it} \times (\Delta y_{it-2})^2)$$
$$\mathbb{E}((\Delta c_{it})^3) \quad \mathbb{E}((\Delta c_{it})^2 \times \Delta c_{it-2}) \quad \mathbb{E}(\Delta c_{it} \times (\Delta c_{it-2})^2)$$
$$\mathbb{E}((\Delta y_{it})^2 \times \Delta c_{it}) \quad \mathbb{E}(\Delta y_{it} \times (\Delta c_{it})^2) \quad \mathbb{E}((\Delta y_{it})^2 \times \Delta c_{it-2})$$
$$\mathbb{E}(\Delta y_{it} \times (\Delta c_{it-2})^2) \quad \mathbb{E}((\Delta y_{it})^2 \times \Delta c_{it+2}) \quad \mathbb{E}(\Delta y_{it} \times \Delta y_{it+2} \times \Delta c_{it})$$
$$\mathbb{E}(\Delta y_{it} \times \Delta y_{it+2} \times \Delta c_{it-2}) \quad \mathbb{E}(\Delta y_{it} \times \Delta y_{it+2} \times \Delta c_{it+2})$$

- $+4^{th}$ moments:

$$\mathbb{E}((\Delta y_{it})^4) \quad \mathbb{E}((\Delta y_{it})^2 \times (\Delta y_{it-2})^2) \quad \mathbb{E}((\Delta y_{it})^3 \times \Delta y_{it-2})$$
$$\mathbb{E}(\Delta y_{it} \times (\Delta y_{it-2})^3) \quad \mathbb{E}((\Delta c_{it})^4) \quad \mathbb{E}((\Delta c_{it})^2 \times (\Delta c_{it-2})^2)$$
$$\mathbb{E}((\Delta c_{it})^3 \times \Delta c_{it-2}) \quad \mathbb{E}(\Delta c_{it} \times (\Delta c_{it-2})^3) \quad \mathbb{E}((\Delta y_{it})^2 \times (\Delta c_{it})^2)$$
$$\mathbb{E}((\Delta y_{it})^3 \times \Delta c_{it}) \quad \mathbb{E}(\Delta y_{it} \times (\Delta c_{it})^3) \quad \mathbb{E}((\Delta y_{it})^2 \times (\Delta c_{it-2})^2)$$
$$\mathbb{E}((\Delta y_{it-2})^2 \times (\Delta c_{it})^2) \quad \mathbb{E}((\Delta y_{it})^3 \times \Delta c_{it-2}) \quad \mathbb{E}(\Delta y_{it} \times (\Delta c_{it-2})^3)$$

- Generalized model targets only highlighted items

A.4
Data

Empirical moments

male earnings & consumption

	Baseline sample		Guvenen et al, 2021
Income data:	PSID		Social Security Admin.
	(1)		(2)
	Panel A. Male earnings growth		
$\mathrm{Var}(\Delta y_{it})$	0.271	(0.010)	0.323
$\mathrm{Skew}(\Delta y_{it})$	-0.569	(0.128)	-1.039
$\mathrm{Kurt}(\Delta y_{it})$	13.711	(0.551)	13.494
	Panel B. Consumption growth		
$\mathrm{Var}(\Delta c_{it})$	0.126	(0.002)	–
$\mathrm{Skew}(\Delta c_{it})$	-0.039	(0.034)	–
$\mathrm{Kurt}(\Delta c_{it})$	4.445	(0.100)	–

- Guvenen et al's income facts prevalent also in our data
  - PSID moments close to administrative moments
  - model revolves around household disposable income, itself exhibits **left skewness** and **very high kurtosis**
- Consumption distribution, however, closer to log-normal

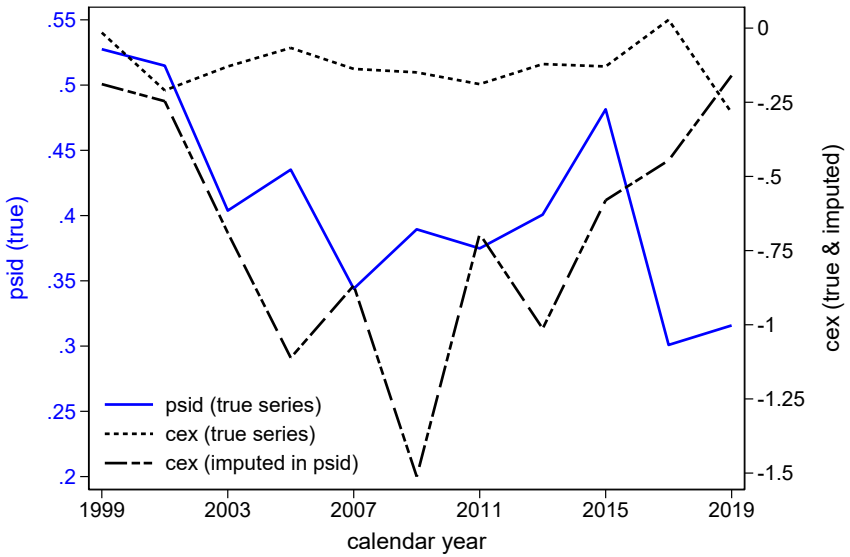
Empirical moments

disposable income & consumption

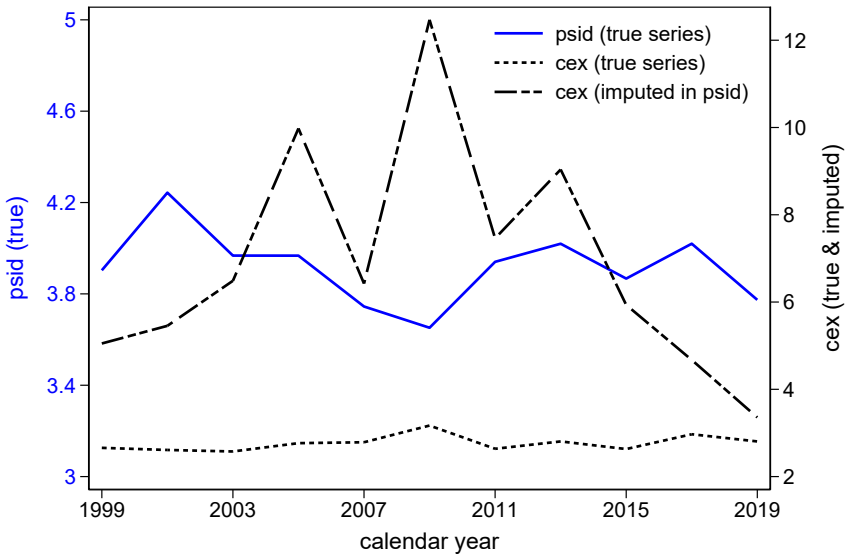
	Baseline sample			
Income data:	PSID		PSID	
Consumption data:	PSID		imputed from CEX	
	(1)		(2)	
Panel A. Moments of income growth				
$\mathrm{Var}(\Delta y_{it})$	0.153	(0.005)	0.153	(0.005)
$\mathrm{Cov}(\Delta y_{it}, \Delta y_{it+1})$	-0.049	(0.002)	-0.049	(0.002)
$\mathrm{Skew}(\Delta y_{it})$	-0.143	(0.098)	-0.143	(0.098)
$\mathrm{Cov}((\Delta y_{it})^2, \Delta y_{it+1})$	0.009	(0.003)	0.009	(0.003)
$\mathrm{Kurt}(\Delta y_{it})$	10.032	(0.509)	10.032	(0.509)
$\mathrm{Cov}((\Delta y_{it})^2, (\Delta y_{it+1})^2)$	0.080	(0.007)	0.080	(0.007)
Panel B. Moments of consumption growth				
$\mathrm{Var}(\Delta c_{it})$	0.126	(0.002)	0.323	(0.015)
$\mathrm{Cov}(\Delta c_{it}, \Delta c_{it+1})$	-0.044	(0.001)	-0.140	(0.009)
$\mathrm{Skew}(\Delta c_{it})$	-0.039	(0.034)	-0.177	(0.083)
$\mathrm{Cov}((\Delta c_{it})^2, \Delta c_{it+1})$	-0.003	(0.001)	0.190	(0.036)
$\mathrm{Kurt}(\Delta c_{it})$	4.445	(0.100)	17.109	(2.111)
$\mathrm{Cov}((\Delta c_{it})^2, (\Delta c_{it+1})^2)$	0.027	(0.001)	0.909	(0.198)
Panel C. Joint moments				
$\mathrm{Cov}(\Delta y_{it}, \Delta c_{it})$	0.008	(0.001)	0.012	(0.002)

Empirical moments

consumption



(a) Skewness of log consumption



(b) Kurtosis of log consumption

- Large discrepancies in higher moments of *log consumption* between PSID and CEX

A.5
Results

Results: linear model with higher moments

Income moments:	2^{nd}	2^{nd}, 3^{rd}	2^{nd}, 3^{rd}, 4^{th}
Consumption data:	PSID	PSID	PSID
	(1)	(2)	(3)
$\phi^{(1)}$ (permanent)	0.152	0.160	0.112
	(0.028)	(0.029)	(0.041)
$\psi^{(1)}$ (transitory)	-0.006	-0.012	0.007
	(0.045)	(0.038)	(0.056)
σ^2_{ξ} (taste heterog.)	0.019	0.019	0.020
	(0.001)	(0.001)	(0.001)
$\sigma^2_{u_c}$ (cons. measur. error)	0.044	0.044	0.044
	(0.002)	(0.002)	(0.001)

- Targeting higher-order moments within the linear consumption model does not change transmission parameters
- If anything, there now seems to be *more* insurance to permanent shocks, a hard-to-comprehend result given severity of tail income risk